Third Quarter Project

Pre-AP Calculus

<u>Instructions</u>: Homework, quizzes, tests, and Supercorrections all provide opportunities for learning mathematics, but nothing beats a project for pulling it all together. Once again, this quarter we will all be working on the same project.

As before, you will be given some class time and some homework time to do your investigation and final write-up. You will use a spreadsheet as you investigate, and your project will be worked on in Google Docs.

Keep in mind that there may be no single correct answer to a question, and you will be evaluated on the basis of your reasoning, justification, and communication skills. **Effective communication of ideas** is a very important component of mathematics.

The **rubric** for evaluation is on the back of this page. Your work will count as one of the two test grades for this unit.

<u>A note on collaboration</u>: You **may** discuss the content of this project with Mr. O'Brien, other students or anyone else but be sure to acknowledge any assistance received. Your final write-up must be your own- any copy and paste from the work of others is unacceptable.

<u>Rough Draft Due</u>: See iCal

<u>Final Draft Due</u>: See iCal

Name:________________________________

Project Rubric

Category	Poor	Fair	Good	Excellent
Presentation (10%) • Is the paper neat? • Is the paper typed? • Is the paper done in an orderly manner? • Is there correct use of grammar and spelling?				
Mathematical precision and completeness (50%) • Are the solutions complete? • Are the solutions correct? • Has there been a correct use of mathematical notation?				
Verbal explanations (30%) • Are the explanations correct? • Are the explanations complete and precise? • Does the project read as a document on its own (i.e. no reference to project question numbers)?				
Graphs (10%) • Are the graphs correct? • Are the graphs neat? • Are the axes labeled properly?				

General Comments:

Final Grade:________

Looking Out to Sea

The Chicago to Mackinac sailboat race is held every summer on Lake Michigan. The residents along the shore of the lake have an opportunity to view the boats as they make their way from Chicago, on the southern end of the lake, to Mackinac Island, on the northern end. One year the winds died down and stranded many of the boats. If you were standing on the shore you could see about twenty boats stalled out in the lake. However, if you climbed up one of the tall bluffs near the shore, your vision was greatly enhanced as almost one hundred boats came into view. Going just a little bit higher dramatically increases how far you can see. Exactly how much farther? That is the question answered in this project.

1. Study Figure 1. The circle represents the circumference of the earth. The line segment labeled a is the distance one is above the surface of the earth. The line segments labeled b are the radii of the earth, and c is the line of sight distance. Since c is tangent to the circle and b is a radius, the angle where they intersect is a right angle.

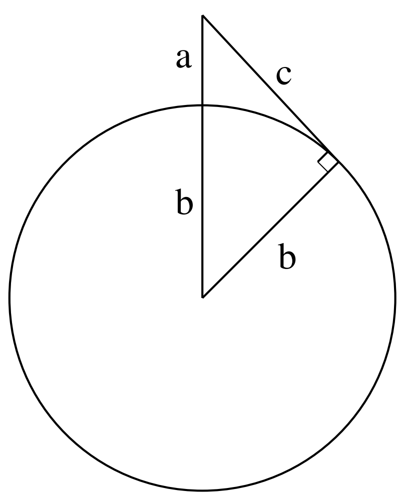


Figure 1

(a) The radius of the earth is about 3950 miles. Using this, find a function whose input is a and whose output is c.

(b) In the function you found in part (a), a and c are in terms of miles. This is a bit cumbersome since we usually don't talk about being so many miles off the surface of the earth. Rewrite your function so that c, your output, is in terms of miles and a, your input, is in terms of feet.[29]

[29] 5280 feet = 1 mile and $5280^2 \text{ft}^2 = 1 \text{ mile}^2$.

2. Let's explore the properties of the function from question 1(b).

(a) Fill in the chart below. The first column is how high you are above the earth. The second column is your line of sight distance. The third column is the difference between your current and previous output divided by the difference between your current and previous input, i.e. an approximate "slope" of the function at that point.

height (feet)	distance (miles)	diff in output/diff in input (miles/feet)
5	2.7352	—
10		
15		
20		

Table 1

(b) Using the information in the chart above, could the graph of this function be a line? Why or why not?

(c) Graph the function you found in question 1(b). What is the relationship between the general shape of the graph and the information given by the numbers in the third column of the chart in question 2(a)?

(d) How high above the earth would you have to be in order to see 20 miles out?

3. The distances we have found so far have been the distance from the observer to the farthest point in a straight line, i.e. the line of sight distance. However, often what people refer to when talking about how far they can see is the distance along the horizon of the earth, i.e. the ground distance. This distance is the arc length, s, in Figure 2 below. Find a formula to express this arc length in terms of a where the input is in miles. Also, find another version of this formula where the input is in feet.

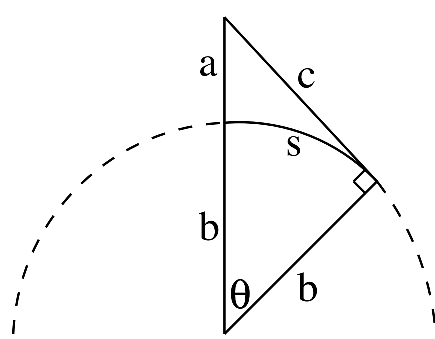


Figure 2

4. We want to compare the line of sight distance to the ground distance along the horizon.

(a) Compare the ground distance to the line of sight distance when a is 10 feet, 1000 feet, 10,000 feet, and 200 miles. What do you notice about the difference between the arc length and the line of sight distance as a gets larger? [Note: The formula commonly used for arc length assumes the angle is measured in *radians*, not degrees. Be sure to set your calculator appropriately.]

height	straight line dist. (miles)	arc length (miles)
10 ft.		
1000 ft.		
10,000 ft.		
200 miles		

Table 2

(b) Using the functions where the input is in miles, graph both the line of sight distance and the ground distance on the same set of axes using a domain of 0 to 500 miles. Then graph using a domain of 0 to 50,000 miles.

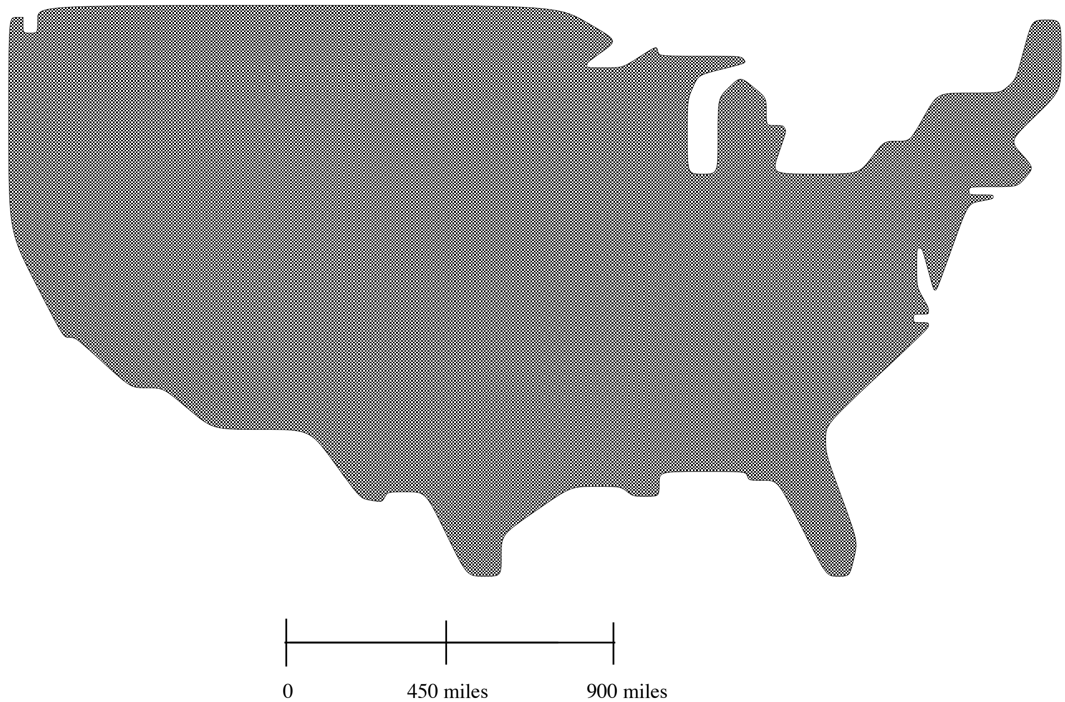
(c) We are interested in exploring what happens to both the line of sight distance and the ground distance as a gets larger. Some functions increase without bound, i.e. keep getting bigger and bigger, and other functions have a limit or upper bound, i.e. a limit as to how large they will get.

i. Think of the physical situation. (Refer back to Figure 2.) The line of sight distance will increase without bound and the ground distance will have a limit. Explain why. What is the limit for the ground distance?

ii. Look at the second graph for question 4(b). Explain how this graph is compatible with your answer to question 4(c)i.

iii. Look at the behavior of your symbolic formulas as a gets large. Explain how this behavior is compatible with your answer to question 4(c)i.

5. The space shuttle orbits approximately 200 miles above the surface of the earth. Is it possible for the someone in the space shuttle to view the entire continental United States? Explain your answer. Use the included map of the United States as a guide.


0
450 miles
900 miles