

A Finely Crafted O'Brien Unit 3 Test

Calculator Section: You may use a calculator. Show all work and circle your answer. Use your time wisely; you will be able to earn additional credit after the timed portion of the test by completing Supercorrections. When you finish, put away your calculator and you can come up to get the non-calculator part- you may continue to work on both sections without your calculator.

1. True or false.

a. $\log_{-2} 4 = 2$

b. $\ln(5 + x) = \ln 5 \ln x$

c. $\log_7 \frac{1}{2} = -\log_7 2$

d. $10^{\log 10^{10}} = 10$

e. $\log x - \log 2 = \frac{\log x}{\log 2}$

f. $\log 4 = \frac{1}{2} \log 16$

g. $\log\left(-\frac{1}{100}\right) = -2$

h. $\log_b 8 = \log_b 8 + \log_b 0$

2. Consider the expression $y = 3 \ln x$. What happens to y if x triples?

3. Explain how the graphs of $y = \log_3 x$ and $y = \log_{1/3} x$ compare.

4. At the right is a “solution” to the equation $100 = 18e^{4k}$.

a. Check the answer $k \approx 0.398$ back in the equation $100 = 18e^{4k}$ and show that it doesn't work.

$$100 = 18e^{4k}$$

$$\ln 100 = \ln(18e^{4k})$$

$$\ln 100 = \ln 18 \ln(e^{4k})$$

$$\frac{\ln 100}{\ln 18} = \ln(e^{4k})$$

$$\frac{\ln 100}{\ln 18} = 4k$$

$$\frac{\ln 100}{4 \ln 18} = k$$

$$k \approx 0.398$$

b. Circle the error in the “solution,” fix that line, and give a correct solution below.

5. Find the smallest value of x for which $3^x \geq 1,000,000$. Is this value a rational number or an irrational number?

6. The toxic radioactive element Plutonium 239 has a half-life of about 24,100 years. How much of a 10 g amount will be left after 1000 years? (The exponential decay model is $y = ae^{-bt}$.)

7. Solve for x .

a. $\log_4 x + \log_4(x-1) = \frac{1}{2}$

b. $e^{2x} = 5$

8. Solve for x .

a. $\ln x + \ln 3 = 1$

b. $e^{2x} - 7e^x + 12 = 0$

9. Here's your chance to show off your log properties!

a. Use the properties of logs to write the expression as a sum, difference, and/or multiple of logs. Simplify where possible.

$$\log_4 \frac{16\sqrt{y}}{x^2 z^4}$$

b. Write the expression as the logarithm of a single quantity.

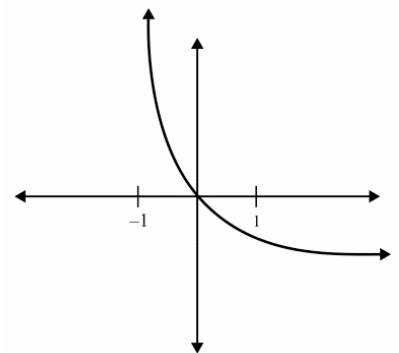
$$\ln 3 + \frac{1}{3} \ln(4-x^2) - \ln x$$

10. Find a positive value of $x \neq 4$ that solves the equation.

$$\frac{\log x}{x} = \frac{\log 4}{4}$$

11. Select integers a and b such that $0 < a^b < 1$. Explain why your selections satisfy the given conditions.

12. Give a logarithmic function that could have the graph at right.



13. Given $f(x) = \ln(x + 3)$. Find the x and y intercepts of $f^{-1}(x)$.

Bonus: Evaluate.

$$9^{\log_3 4} + 4^{\log_3 9}$$